FERMILAB–PUB–95/236–T

OSU Preprint 306

July 1995

Construction of an $SO(10) \times U(1)_F$ Model of the Yukawa Interactions

Carl H. ALBRIGHT

Department of Physics, Northern Illinois University, DeKalb, Illinois 60115*

and

Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, Illinois 60510†

Satyanarayan NANDI

Department of Physics, Oklahoma State University, Stillwater, Oklahoma 74078‡

Abstract

We construct a supersymmetric $SO(10)\times U(1)_F$ model of the Yukawa interactions at the grand unification scale from knowledge of a phenomenological set of mass matrices obtained by a previous bottom-up approach. The $U(1)_F$ family symmetry determines the textures for the Majorana and generic Dirac mass matrices, while the $SO(10)$ symmetry relates each particular element of the up, down, neutrino and charged lepton Dirac matrices. The dominant second and third family contributions in the Dirac sector are renormalizable, while the remaining contributions to the Dirac mass matrices are of higher order, restricted by the $U(1)_F$ family symmetry to a small set of tree diagrams, and mainly complex-symmetric. The tree diagrams for the Majorana mass matrix are all non-renormalizable and of progressively higher-order, leading to a nearly

*Permanent address

†Electronic address: ALBRIGHT@FNALV

‡Electronic address: PHYSSNA@OSUCC

geometrical structure. Pairs of $\mathbf{1}, \mathbf{45}, \mathbf{10}$ and $\mathbf{126}$ Higgs representations enter with those having large vacuum expectation values breaking the symmetry down to $SU(3)_c \times SU(2)_L \times U(1)_Y$ near the grand unification scale. In terms of 12 parameters expressed as the Yukawa couplings times vacuum expectation values for the Higgs representations employed, a realistic set of 15 quark and lepton masses (including those for the 3 heavy righthanded Majorana neutrinos)and 8 mixing parameters emerges for the neutrino scenario involving the non-adiabatic conversion of solar neutrinos and the depletion of atmospheric muon-neutrinos through oscillations into tau-neutrinos.

PACS numbers: 12.15.Ff, 12.60Jv

I. INTRODUCTION

The standard model (SM) of strong and electroweak interactions, while providing excellent agreement with experiment todate, is known to be woefully inadequate to explain the mass spectrum and mixings of the three families of quarks and leptons. One needs to go beyond the standard model in order to relate the independent Yukawa couplings to each other. Of the various possibilities, supersymmetric grand unified theories and superstring theories seem to hold the most promise for successfully incorporating the Yukawa interactions in a more satisfactory fashion. In this paper we shall restrict our attention to supersymmetric $SO(10)$ grand unification, which has been shown [1] to unify the gauge couplings successfully at a scale of $\Lambda_{SGUT} \sim 10^{16}$ GeV.

It is a generally held opinion that knowledge of the mass matrices in the weak flavor basis can provide insights into the dynamical mass-generating mechanism.[2] This follows from the fact that the mass eigenvalues are obtained by diagonalization of the mass matrices, while the mixing matrices in the mass eigenbasis can be constructed from knowledge of the diagonalizing matrices connecting the two bases. By starting from the correct mass matrices, one should then be able to deduce the observed quark and lepton masses and mixings after evolving the results down to the present "low energy" scales.

Generally two procedures are at one's disposal for the identification of the "correct" mass matrices. One can attempt to postulate a particular structure or "texture" for the mass matrices based on some well-defined and presumably simple theoretical concepts such as the unification group and/or the number of texture zeros present.[3] This procedure has been employed by most researchers in the past twenty years. Alternatively, one can make use of the known low energy mass and mixing data, supplemented by reasonable guesses for data which is not yet well determined, in order to extract mass matrices within some framework at the unification scale which yield the low energy data in question. Of special

interest are neutrino scenarios incorporating the Mikheyev - Smirnov - Wolfenstein (MSW) [4] nonadiabatic resonant conversion interpretation of the depletion of solar electron-neutrinos [5] and either the observed depletion of atmospheric muon-neutrinos through oscillations [6] or neutrinos of satisfactory mass to contribute to the hot component of mixed dark matter [7], for example.

In a series of papers [8] the authors have demonstrated the latter "bottom-up" approach by making use of Sylvester's theorem [9] to construct mass matrices from the low energy masses and mixing matrices evolved to the unification scale. In doing so, we have attempted to look for simplicity of the mass matrices in the $SO(10)$ framework while varying the quark and lepton weak bases. Such simplicity was found for the MSW solar and atmospheric neutrino depletions in the bases where the up quark and Dirac neutrino mass matrices are real and diagonal, while the down quark and charged lepton matrices are in general complex symmetric. The right-handed Majorana neutrino matrix exhibits a simple nearly geometrical texture.

From the phenomenological mass matrices constructed, we have attempted to derive mass matrices of similar textures from some well-defined family symmetry. In particular, we find within an $SO(10) \times U(1)_F$ symmetry framework that we can reproduce all the known and assumed-known low energy mass and mixing data for the quarks and leptons in terms of products of Yukawa couplings and Higgs vacuum expectation values (VEVs). The $U(1)_F$ symmetry controls the textures for the generic Dirac and Majorana mass matrices, while $SO(10)$ relates particular elements of the up, down, Dirac neutrino and charged lepton mass matrices to each other.

In this paper we shall present all the details for this model construction which were summarized in a short letter submitted elsewhere [10]. Section II summarizes the bottom-up procedure and the phenomenological mass matrices obtained for the neutrino scenario

preferred. The $U(1)_F$ family symmetry is introduced and applied in the Dimopoulos tree-diagram approach [11] in Sect. III for the contributions to the mass matrices. In Sect. IV the diagramatic contributions to the mass matrix elements are explicitly given with quantitative results presented in Sect V. Our work is summarized in Sect. VI.

II. PHENOMENOLOGICAL MATRICES from a BOTTOM-UP APPROACH

We begin by presenting the low scale input and procedure by which we were able to construct a relatively simple $SO(10)$ set of phenomenological mass matrices at the SUSY GUT scale as spelled out in detail in Ref. [8] The relevant framework is assumed to be that of SUSY $SO(10)$ grand unification at a scale of $\Lambda_{SGUT} = 1.2 \times 10^{16}$ GeV with supersymmetry breaking occuring at a scale of 180 GeV, in order that we can use the analytical one-loop evolution formulas and results given by Naculich [12].

For the low scale quark data, we assumed the following set of quark masses and Cabbibo - Kobayashi - Maskawa (CKM) mixing matrix [13]

$$\begin{aligned} m_u(1\text{GeV}) &= 5.1 \text{ MeV}, & m_d(1\text{GeV}) &= 8.9 \text{ MeV} \\ m_c(m_c) &= 1.27 \text{ GeV}, & m_s(1\text{GeV}) &= 175 \text{ MeV} \\ m_t(m_t) &= 150 \text{ GeV}, & m_b(m_b) &\simeq 4.25 \text{ GeV} \end{aligned} \tag{2.1a}$$

$$V_{CKM} = \begin{pmatrix} 0.9753 & 0.2210 & 0.0031 e^{-i155^o} \\ -0.2206 & 0.9744 & 0.043 \\ \begin{matrix} 0.011 \\ \quad -0.001i \end{matrix} & -0.041 & 0.999 \end{pmatrix} \tag{2.1b}$$

The light quark masses were chosen to be the central values given by Gasser and Leutwyler [14], while the heavy physical top mass was set equal to 160 GeV prior to its discovery yielding a running mass of 150 GeV. We assumed a value of 0.043 for V_{cb}, which is now thought to be closer to 0.040, and applied strict unitarity to determine V_{ub}, V_{td} and V_{ts}.

The greatest $SO(10)$ simplicity was obtained for the neutrino scenario incorporating the observed depletion of solar neutrinos [5] through the nonadiabatic MSW [4] matter conversion of electron-neutrinos into muon-neutrinos in the interior of the sun and the depletion of atmospheric muon-neutrinos through oscillation into tau-neutrinos observed now by several deep mine collaborations [6]. The central values deduced for these mixing plane results are

$$\begin{aligned} \delta m^2_{12} &\sim 5 \times 10^{-6}\ \mathrm{eV}^2, \qquad & \sin^2 2\theta_{12} &\sim 0.008 \\ \delta m^2_{23} &\sim 1 \times 10^{-2}\ \mathrm{eV}^2, \qquad & \sin^2 2\theta_{23} &\sim 0.9 \end{aligned} \tag{2.2}$$

We took for the lepton input

$$\begin{aligned} m_{\nu_e} &= 0.5 \times 10^{-6}\ \mathrm{eV}, \qquad & m_e &= 0.511\ \mathrm{MeV} \\ m_{\nu_\mu} &= 0.224 \times 10^{-2}\ \mathrm{eV}, \qquad & m_\mu &= 105.3\ \mathrm{MeV} \\ m_{\nu_\tau} &= 0.105\ \mathrm{eV}, \qquad & m_\tau &= 1.777\ \mathrm{GeV} \end{aligned} \tag{2.3a}$$

and

$$V_{lept} = \begin{pmatrix} 0.9990 & 0.0447 & 0.0076e^{-i155^o} \\ -0.0363 & 0.8170 & 0.575 \\ \begin{matrix} 0.026 \\ -0.007i \end{matrix} & -0.570 & 0.818 \end{pmatrix} \tag{2.3b}$$

These masses and mixing matrix data were evolved to the SUSY GUT scale by using formulas given by Naculich [12] as spelled out in detail in Ref. 8. We could then reconstruct complex-symmetric mass matrices at the SUSY GUT scale by making use of Sylvester's theorem [9] as illustrated by Kusenko [15] for the quark sector. The construction is not unique, for one is free to change the quark and lepton weak bases by letting two parameters, x_q and x_ℓ, vary independently over their support regions, $0 \leq x \leq 1$. For x_q $(x_\ell) = 0$, the up quark (Dirac neutrino) mass matrix is diagonal; while for x_q $(x_\ell) = 1$, the down quark (charged lepton) mass matrix is diagonal. One is also free to vary the signs of the mass eigenvalues.

By varying the signs of the mass eigenvalues and the two parameters x_q and x_ℓ, we then searched for a simple $SO(10)$ structure for the mass matrices. The greatest simplicity

occurred with $x_q = 0$ and $x_\ell = 0.93$ corresponding to diagonal up quark and Dirac neutrino mass matrices and leading to

$$M^U \sim M^{N_{Dirac}} \sim diag(\overline{126};\ \overline{126};\ 10) \tag{2.4a}$$

$$M^D \sim M^E \sim \begin{pmatrix} 10', \overline{126} & 10', \overline{126}' & 10' \\ 10', \overline{126}' & \overline{126} & 10' \\ 10' & 10' & 10 \end{pmatrix} \tag{2.4b}$$

with $M^D_{11},\ M^E_{12}$ and M^E_{21} anomalously small and only the 13 and 31 elements complex. Entries in the matrices stand for the Higgs representations contributing to those elements, which we elaborate upon in the next Section. We have assumed complete unification for the Yukawa couplings of the third families of quarks and leptons and that vacuum expectation values (VEVs) develop only for the symmetric representations **10** and **126**. The **10**'s contribute equally to $(M^U,\ M^D)$ and $(M^{N_{Dirac}},\ M^E)$, while the **126**'s weight $(M^U,\ M^D)$ and $(M^{N_{Dirac}},\ M^E)$ in the ratio of 1 : -3. The Majorana neutrino mass matrix M^R, determined from the seesaw formula [16] with use of $M^{N_{Dirac}}$ and the reconstructed light neutrino mass matrix, exhibits a nearly geometrical structure [17] given by

$$M^R \sim \begin{pmatrix} F & -\sqrt{FE} & \sqrt{FC} \\ -\sqrt{FE} & E & -\sqrt{EC} \\ \sqrt{FC} & -\sqrt{EC} & C \end{pmatrix} \tag{2.4c}$$

where $E \simeq \frac{5}{6}\sqrt{FC}$ with all elements relatively real [18]. It can not be purely geometrical, however, since the singular rank-1 matrix can not be inverted as required by the seesaw formula, $M^{N_{eff}} \simeq -M^{N_{Dirac}}(M^R)^{-1}M^{N^T_{Dirac}}$.

III. $U(1)_F$ FAMILY SYMMETRY and RESULTING TREE DIAGRAMS

The challenge is now to introduce a family symmetry which will enable us to derive the mass matrix textures derived above phenomenologically from our bottom-up approach. For

this purpose, we propose to use a $U(1)_F$ family symmetry [19], where we leave open for the time being whether the symmetry is global or local in which case it can be gauged. Before proceeding with this, we review briefly the elements of the $SO(10)$ symmetry group which play important roles in our model construction.

In the $SO(10)$ framework, each family of left-handed quarks, leptons, conjugate quarks and conjugate leptons belongs to a **16** dimensional representation. It is convenient to represent a given flavor (and color) member of the ith family and its conjugate by the two components $\Psi_{iL} = (\psi_{iL}, (\psi^c)_{iL})$. In the corresponding three-family basis ordered as follows, $\Psi_L = \{\psi_{1L}, \psi_{2L}, \psi_{3L}, (\psi^c)_{1L}, (\psi^c)_{2L}, (\psi^c)_{3L}\}$, the contributions to the up or down quark, neutrino or charged lepton, mass terms in the Yukawa Lagrangian are then given by

$$\mathcal{L} = \Psi_L^T C^{-1} \mathcal{M} \Psi_L + \text{h.c.} \tag{3.1a}$$

where the 6 x 6 matrix can be written in terms of 3 x 3 submatrices

$$\mathcal{M} = \mathcal{M}^{\mathcal{T}} = \begin{pmatrix} M^L & M_{Dirac} \\ M_{Dirac}^T & M^R \end{pmatrix} \tag{3.1b}$$

with the individual contributions referring to

$$\begin{aligned} M^L : &\quad (\psi_{iL})^T C^{-1} \psi_{jL} \\ M_{Dirac} : &\quad (\psi_{iL})^T C^{-1} (\psi^c)_{jL} = \overline{\psi_{jR}} \psi_{iL} \\ M_{Dirac}^T : &\quad (\psi^c)_{iL}{}^T C^{-1} \psi_{jL} = \overline{\psi_{iR}} \psi_{jL} \\ M^R : &\quad (\psi^c)_{iL}^T C^{-1} (\psi^c)_{jL} = \overline{\psi_{iR}} (\psi^c)_{jL} \end{aligned} \tag{3.1c}$$

Here the diagonal block entries appear only for neutrinos with M^L the left-handed Majorana neutrino mass matrix which we take to vanish, while M^R is the right-handed Majorana neutrino mass matrix which receives large contributions near the SUSY GUT scale.

By construction the 6 x 6 matrix $\mathcal{M}$ is complex symmetric, but the Dirac mass submatrix is not necessarily complex symmetric. We shall assume that the dominant contributions are

complex symmetric and that any departures from this form arise from small higher-order corrections. Recall that the $SO(10)$ product rules read

$$\mathbf{16} \times \mathbf{16} \ = \mathbf{10}_s + \mathbf{120}_a + \mathbf{126}_s \tag{3.2a}$$

$$\mathbf{16} \times \overline{\mathbf{16}} \ = \mathbf{1} + \mathbf{45} + \mathbf{210} \tag{3.2b}$$

Hence we shall assume that only the symmetric Higgs representations **10** and **126** develop low scale VEVs, while the antisymmetric **120** does not. In terms of the $SU(5)$ decompositions, we have

$$\mathbf{10} \rightarrow 5 + \bar{5}, \qquad \mathbf{126} \rightarrow \overline{50} + 45 + \overline{15} + 10 + \bar{5} + 1 \tag{3.3a}$$

The up-type quarks and Dirac neutrinos then can receive contributions from the neutral members of $\mathbf{10}(5)$ and $\overline{\mathbf{126}}(5)$, the down-type quarks and charged leptons from those of $\mathbf{10}(\bar{5})$ and $\overline{\mathbf{126}}(\overline{45})$, and the heavy right-handed Majorana neutrinos from those of $\overline{\mathbf{126}}(1)$. We shall later assume the Higgs representations **1** and **45** play a role in the higher-order corrections, where the $\mathbf{45}(1)$ and $\mathbf{45}(24)$ develop VEVs according to the decomposition

$$\mathbf{45} \rightarrow 24 + 10 + \overline{10} + 1 \tag{3.3b}$$

Returning to the phenomenological mass matrices obtained in Section II, we use the textures given in (2.4a,b,c) as our starting point for the construction of an $SO(10) \times U(1)_F$ model of the Yukawa interactions. We find it useful to introduce a generic Dirac matrix, M_{Dirac}, to go along with the one Majorana matrix, M^R. The $U(1)_F$ family symmetry will then determine the textures for M_{Dirac} and M^R, while the $SO(10)$ symmetry will relate the corresponding matrix elements of the four Dirac matrices M^U, M^D, M^N and M^E to each other.

Simplicity of the $SO(10)$ structure requires that just one Higgs **10** representation contributes to the $(M_{Dirac})_{33}$ element (hereafter labeled D33). Since a **10** contributes equally to

the 33 elements of all four Dirac matrices, this implies that we assume complete unification of the Yukawa couplings at the unification scale: $\bar{m}_\tau = \bar{m}_b = \bar{m}_t/\tan\beta_{10}$, where $\tan\beta_{10}$ is equal to the ratio of the up quark to the down quark VEVs in the **10**, i.e.,

$$\begin{aligned} \bar{m}_t &= g_{10}(v/\sqrt{2})\sin\beta_{10} \equiv g_{10}v_u \\ \bar{m}_b = \bar{m}_\tau &= g_{10}(v/\sqrt{2})\cos\beta_{10} \equiv g_{10}v_d \\ \tan\beta_{10} &= v_u(5)/v_d(\bar{5}) \end{aligned} \tag{3.4a}$$

in terms of the $SU(5)$ decomposition of $SO(10)$ with $v = 246$ GeV. The same **10** can not contribute to D23 = D32, for the diagonal nature of M^U and M^N requires the presence of another $\mathbf{10}'$ with

$$\tan\beta_{10'} = v'_u(5')/v'_d(\bar{5}') = 0 \tag{3.4b}$$

Likewise we assume a pure $\overline{\mathbf{126}}$ contribution to D22 with

$$\tan\beta_{\overline{126}} = w_u(5)/w_d(\overline{45}) \tag{3.4c}$$

In other words, for simplicity we have taken the 2-3 sector of M_{Dirac} to be renormalizable with two **10**'s and one $\overline{\mathbf{126}}$ developing low scale VEVs. We illustrate the renormalizable 3-point tree diagrams in Fig. 1a.

We now assign $U(1)_F$ charges as follows to the three families (in order of appearance) and to the three Higgs representations introduced which generate low scale VEVs with the numerical values to be determined later:

$$\mathbf{16}_3^\alpha,\ \mathbf{16}_2^\beta,\ \mathbf{16}_1^\gamma,\ \mathbf{10}^a,\ \mathbf{10}'^b,\ \overline{\mathbf{126}}^{\mathbf{c}} \tag{3.5a}$$

Conservation of $U(1)_F$ charges then requires $2\alpha + a = 0,\ \alpha + \beta + b = 0$ and $2\beta + c = 0$ as seen from the diagrams in Fig. 1a.

We assume the rest of the M_{Dirac} elements arise from higher-order tree diagrams as first suggested by Dimopoulos [11] twelve years ago. The point is that not only does SUSY

control the running of the Yukawa couplings between the SUSY GUT scale and the weak scale where it is assumed to be softly broken, but it also allows one to assume that only simple tree diagrammatic contributions to the mass matrices need be considered as a result of the non-renormalization theorem [20] applied to loop diagrams. While the low-scale VEVs introduced act only once in each diagram, other GUT scale VEVs arising from **1** and **45** Higgs representations can connect superheavy GUT scale **16** fermions and their conjugate $\overline{\mathbf{16}}$ mirrors to each other and to the three light **16** families. The superheavy fermions and their mirrors pair off and get masses slightly higher than the SUSY GUT breaking scale, so their propagators in the higher-order tree diagrams lead to non-renormalizable contributions scaled by their masses.

For each **45** Higgs representation, as noted earlier in terms of the SU(5) decomposition given in (3.3b), VEVs can develop in the orthogonal directions

$$< \mathbf{45}_X > \sim \mathbf{45}(1), \qquad < \mathbf{45}_Y > \sim \mathbf{45}(24) \tag{3.6a}$$

or in any non-orthogonal directions. One such direction of interest corresponds to the hypercharge direction for flipped [21] $SU(5) \times U(1)$ as clarified in Table I:

$$< \mathbf{45}_Z > = \frac{6}{5} < \mathbf{45}_X > - \frac{1}{5} < \mathbf{45}_Y > \tag{3.6b}$$

While the $< \mathbf{45}_X >$ VEV breaks $SO(10) \rightarrow SU(5)$, the $< \mathbf{45}_Z >$ VEV breaks $SO(10) \rightarrow$ flipped $SU(5)$. Alternatively, if the $SO(10) \rightarrow SU(5)$ breaking occurs earlier by some other VEV such as $< \overline{126}' >$ as required later for the Majorana sector, the combined action of $< \mathbf{45}_X >$ and $< \mathbf{45}_Z >$ will result in the breaking of $SU(5) \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y$.

Since the D13 and D23 elements in (2.4a,b) have the same $\mathbf{10}'$ transformation property, this suggests that we introduce a $\mathbf{45}^e_X$ Higgs field and construct an explicitly complex-symmetric dimension-6 tree diagram as shown in Fig. 1b, for which $U(1)_F$ charge conservation requires $\alpha + \gamma + b + 2e = 0$. We shall later give the four Dirac mass matrix contributions

derived from D13 by use of Table I which confirms that D13 and D23 do have the same $\mathbf{10'}$ transformation property, i.e., the contributions to M^U and M^N vanish while those to M^D and M^E are non-zero and equal.

The D12 element, on the other hand, appears to arise from a linear combination of $\mathbf{10'}$ and new $\overline{\mathbf{126}}$ VEV contributions for which $(M^E)_{12} \ll (M^D)_{12}$. Rather than introduce another new renormalizable diagram, we can make use of the fact that a $\mathbf{45}_Z$ Higgs develops a VEV which vanishes for the charged lepton D12 diagram as seen from Table I. We then introduce a new $\mathbf{45}^h_Z$ Higgs field and construct the complex-symmetric dimension-6 tree diagram shown in Fig. 1b. Note that detailed study showed that to reduce the number of contributing diagrams the $\mathbf{10'}$ Higgs line should leave the diagram, or equivalently, the $\mathbf{10'^*}$ line should enter the diagram, so $U(1)_F$ charge conservation requires $\beta + \gamma - b + 2h = 0$.

The D11 element is dimension-8 or higher, and we leave it unspecified. The complex-symmetric leading-order Yukawa diagrams which we wish to generate are then neatly summarized by the ordering of the Higgs fields where all external lines enter the diagrams:

$$\begin{aligned}
D33 &: \ \mathbf{16}_3 - \mathbf{10} - \mathbf{16}_3 \\
D23 &: \ \mathbf{16}_2 - \mathbf{10'} - \mathbf{16}_3 \\
D32 &: \ \mathbf{16}_3 - \mathbf{10'} - \mathbf{16}_2 \\
D22 &: \ \mathbf{16}_2 - \overline{\mathbf{126}} - \mathbf{16}_2 \\
D13 &: \ \mathbf{16}_1 - \mathbf{45}_X - \mathbf{10'} - \mathbf{45}_X - \mathbf{16}_3 \\
D31 &: \ \mathbf{16}_3 - \mathbf{45}_X - \mathbf{10'} - \mathbf{45}_X - \mathbf{16}_1 \\
D12 &: \ \mathbf{16}_1 - \mathbf{45}_Z - \mathbf{10'^*} - \mathbf{45}_Z - \mathbf{16}_2 \\
D21 &: \ \mathbf{16}_2 - \mathbf{45}_Z - \mathbf{10'^*} - \mathbf{45}_Z - \mathbf{16}_1
\end{aligned} \tag{3.7a}$$

In order to obtain a different set of diagrams and hence a different texture for the Majorana matrix, we begin the M33 contribution with a dimension-6 diagram shown in Fig. 1c by including a new $\overline{\mathbf{126}}'^d$ Higgs which develops a VEV at the GUT scale in the $SU(5)$ singlet

direction, along with a pair of $\mathbf{1}^g$ Higgs fields. Here $2\alpha + d + 2g = 0$. The nearly geometric structure [8] for M^R can then be generated by appending more Higgs fields to each diagram. For M23 we introduce another $\mathbf{1}'^f$ Higgs field to construct a diagram with one $\overline{\mathbf{126}}'^d$, one $\mathbf{45}^e_X$, one $\mathbf{1}'^f$ and two $\mathbf{1}^g$ fields with charge conservation demanding $\alpha + \beta + d + 2g + e + f = 0$. The new $\mathbf{1}'$ field is needed in order to scale properly the Majorana matrix elements relative to each other. The remaining leading-order diagrams of the complex-symmetric Majorana mass matrix follow by appending more $\mathbf{45}^e_X$, $\mathbf{45}^h_Z$ and $\mathbf{1}'^f$ Higgs lines. The pattern is made clear from the charge conservation equations: $2\beta + d + 2g + 2e + 2f = 0$ for M22, $\alpha + \gamma + d + 2g + e + h + 2f = 0$ for M13, $\beta + \gamma + d + 2g + 2e + h + 3f = 0$ for M12, and $2\gamma + d + 2g + 2e + 2h + 4f = 0$ for M11.

In summary, the following Higgs representations have been introduced in addition to those in (3.5a):

$$\overline{\mathbf{126}}'^d,\ \mathbf{45}^e_X,\ \mathbf{45}^h_Z,\ \mathbf{1}^g,\ \mathbf{1}'^f \tag{3.5b}$$

all of which generate massive VEVs near the GUT scale. In order to obtain CP-violation in the quark and lepton mixing matrices, we allow the VEVs for $\mathbf{45}_X$, $\mathbf{45}_Z$, $\mathbf{1}$ and $\mathbf{1}'$ to be complex, but the VEVs associated with the $\mathbf{10}$, $\mathbf{10}'$,$\overline{\mathbf{126}}$ and $\overline{\mathbf{126}}'$ representations can be taken to be real without loss of generality as seen from our bottom-up results. Clearly, many permutations of the Higgs fields are possible in the higher-order diagrams.

At this point a computer search was carried out to generate $U(1)_F$ charge assignments leading to the fewest additional diagrams allowed by charge conservation. An especially interesting charge assignment stood out for which

$$\begin{aligned}&\alpha = 9,\ \beta = -1,\ \gamma = -8\\ &a = -18,\ b = -8,\ c = 2,\ d = -22,\ e = 3.5,\ f = 6.5,\ g = 2.0,\ h = 0.5\end{aligned} \tag{3.8a}$$

One should note that since $\alpha + \beta + \gamma = 0$, the $[SO(10)]^2 \times U(1)_F$ triangle anomaly vanishes, whereas the $[U(1)_F]^3$ anomaly does not. Simplicity then suggests that the $U(1)_F$ family sym-

metry group can be global with a familon being generated upon its breaking. Alternatively, the $U(1)_F$ group can be local and gauged if the $[U(1)_F]^3$ anomaly is canceled by the addition of several $SO(10)$-singlet fermions with appropriate $U(1)$ charges, or perhaps better still, by the Green-Schwarz mechanism [22] provided the model can be derived from string theory. We intend to study this point at greater length elsewhere and do not commit ourselves here to either possibility.

With the above charge assignments we can further greatly limit the number of permutations and eliminate other unwanted diagrams by restricting the $U(1)_F$ charges appearing on the superheavy internal fermion lines. With the following minimum set of allowed charges for the left-handed superheavy fermions F_L and their mirror partners F_L^c

$$\begin{array}{lccccccccc} F_L: & -0.5, & 1.0, & 2.0, & 4.0, & 4.5, & -4.5, & -7.5, & 11.0, & 12.5 \\ F_L^c: & 0.5, & -1.0, & -2.0, & -4.0, & -4.5, & 4.5, & 7.5, & -11.0, & -12.5 \end{array} \tag{3.8b}$$

as determined by another computer program, we recover just the leading-order diagrams listed in (3.7a) for the generic Dirac mass matrix together with the following uniquely-ordered diagrams for the complex-symmetric Majorana mass matrix

$$\begin{array}{ll} M33: & \mathbf{16}_3 - \mathbf{1} - \overline{\mathbf{126}}' - \mathbf{1} - \mathbf{16}_3 \\ M23: & \mathbf{16}_2 - \mathbf{1} - \mathbf{45}_X - \mathbf{1}' - \overline{\mathbf{126}}' - \mathbf{1} - \mathbf{16}_3 \\ M32: & \mathbf{16}_3 - \mathbf{1} - \overline{\mathbf{126}}' - \mathbf{1}' - \mathbf{45}_X - \mathbf{1} - \mathbf{16}_2 \\ M22: & \mathbf{16}_2 - \mathbf{1} - \mathbf{45}_X - \mathbf{1}' - \overline{\mathbf{126}}' - \mathbf{1}' - \mathbf{45}_X - \mathbf{1} - \mathbf{16}_2 \\ M13: & \mathbf{16}_1 - \mathbf{45}_X - \mathbf{1}' - \mathbf{1} - \mathbf{45}_Z - \mathbf{1}' - \overline{\mathbf{126}}' - \mathbf{1} - \mathbf{16}_3 \\ M31: & \mathbf{16}_3 - \mathbf{1} - \overline{\mathbf{126}}' - \mathbf{1}' - \mathbf{45}_Z - \mathbf{1} - \mathbf{1}' - \mathbf{45}_X - \mathbf{16}_1 \\ M12: & \mathbf{16}_1 - \mathbf{45}_X - \mathbf{1}' - \mathbf{1} - \mathbf{45}_Z - \mathbf{1}' - \overline{\mathbf{126}}' - \mathbf{1}' - \mathbf{45}_X - \mathbf{1} - \mathbf{16}_2 \\ M21: & \mathbf{16}_2 - \mathbf{1} - \mathbf{45}_X - \mathbf{1}' - \overline{\mathbf{126}}' - \mathbf{1}' - \mathbf{45}_Z - \mathbf{1} - \mathbf{1}' - \mathbf{45}_X - \mathbf{16}_1 \\ M11: & \mathbf{16}_1 - \mathbf{45}_X - \mathbf{1}' - \mathbf{1} - \mathbf{45}_Z - \mathbf{1}' - \overline{\mathbf{126}}' - \mathbf{1}' - \mathbf{45}_Z - \mathbf{1} - \mathbf{1}' - \mathbf{45}_X - \mathbf{16}_1 \end{array} \tag{3.7b}$$

Several other higher-order diagrams are allowed by the $U(1)_F$ charges given in (3.8a,b) and

appear for D11, D22, M23 and M32 with the Higgs fields ordered as follows:

$$\begin{aligned}
D11:&\quad \mathbf{16}_1 - \mathbf{45}_X - \mathbf{1}' - \mathbf{1} - \mathbf{10}' - \mathbf{1} - \mathbf{1}' - \mathbf{45}_X - \mathbf{16}_1\\
D22:&\quad \mathbf{16}_2 - \mathbf{45}_Z - \mathbf{10}'^* - \mathbf{1}'^* - \mathbf{16}_2, \quad \mathbf{16}_2 - \mathbf{1}'^* - \mathbf{10}'^* - \mathbf{45}_Z - \mathbf{16}_2\\
M23:&\quad \mathbf{16}_2 - \mathbf{45}^*_X - \mathbf{1}' - \mathbf{1} - \mathbf{45}_Z - \mathbf{1}' - \overline{\mathbf{126}}' - \mathbf{1} - \mathbf{16}_3\\
M32:&\quad \mathbf{16}_3 - \mathbf{1} - \overline{\mathbf{126}}' - \mathbf{1}' - \mathbf{45}_Z - \mathbf{1} - \mathbf{1}' - \mathbf{45}^*_X - \mathbf{16}_2
\end{aligned} \tag{3.7c}$$

These corrections to M23 and M32 ensure that M^R is rank 3 and nonsingular, so that the seesaw formula [16] can be applied. Up to this point the contributions are all complex-symmetric.

Additional correction terms of higher order which need not be complex-symmetric can be generated for the Dirac and Majorana matrix elements, if one allows additional superheavy fermion pairs with new $U(1)_F$ charges. Such a subset which does not destroy the pattern constructed above, but helps to improve the numerical results for the lepton masses and mixings, consists of the following:

$$\begin{aligned}
F_L:&\quad 1.5,\quad -6.0,\quad -6.5\\
F^c_L:&\quad -1.5,\quad 6.0,\quad 6.5
\end{aligned} \tag{3.8c}$$

The additional diagrams arising from this expanded set of superheavy fermions are:

$$\begin{aligned}
\text{D11:}&\quad \mathbf{16}_1 - \mathbf{1} - \overline{\mathbf{126}} - \mathbf{1} - \mathbf{1}' - \mathbf{45}_X - \mathbf{16}_1, \qquad \mathbf{16}_1 - \mathbf{45}_X - \mathbf{1}' - \mathbf{1} - \overline{\mathbf{126}} - \mathbf{1} - \mathbf{16}_1\\
\text{D11:}&\quad \mathbf{16}_1 - \mathbf{1} - \mathbf{45}^*_Z - \mathbf{1} - \mathbf{1}' - \mathbf{1} - \overline{\mathbf{126}} - \mathbf{1} - \mathbf{16}_1,\\
&\qquad \mathbf{16}_1 - \mathbf{1} - \overline{\mathbf{126}} - \mathbf{1} - \mathbf{1}' - \mathbf{1} - \mathbf{45}^*_Z - \mathbf{1} - \mathbf{16}_1\\
\text{D11:}&\quad \mathbf{16}_1 - \mathbf{1} - \mathbf{45}^*_Z - \mathbf{1} - \mathbf{1}' - \mathbf{1} - \mathbf{10}' - \mathbf{1} - \mathbf{1}' - \mathbf{45}_X - \mathbf{16}_1,\\
&\qquad \mathbf{16}_1 - \mathbf{45}_X - \mathbf{1}' - \mathbf{1} - \mathbf{10}' - \mathbf{1} - \mathbf{1}' - \mathbf{1} - \mathbf{45}^*_Z - \mathbf{1} - \mathbf{16}_1\\
\text{D12:}&\quad \mathbf{16}_1 - \mathbf{1} - \overline{\mathbf{126}} - \mathbf{1} - \mathbf{1}' - \mathbf{45}^*_X - \mathbf{16}_2\\
\text{D21:}&\quad \mathbf{16}_2 - \mathbf{45}^*_X - \mathbf{1}' - \mathbf{1} - \overline{\mathbf{126}} - \mathbf{1} - \mathbf{16}_1\\
\text{D12:}&\quad \mathbf{16}_1 - \mathbf{1} - \overline{\mathbf{126}} - \mathbf{45}^*_Z - \mathbf{45}_X - \mathbf{1} - \mathbf{16}_2\\
\text{D21:}&\quad \mathbf{16}_2 - \mathbf{1} - \mathbf{45}_X - \mathbf{45}^*_Z - \overline{\mathbf{126}} - \mathbf{1} - \mathbf{16}_1\\
\text{D12:}&\quad \mathbf{16}_1 - \mathbf{1} - \mathbf{45}^*_Z - \overline{\mathbf{126}} - \mathbf{45}_X - \mathbf{1} - \mathbf{16}_2
\end{aligned} \tag{3.7d}$$

D21: $\mathbf{16}_2 - \mathbf{1} - \mathbf{45}_X - \overline{\mathbf{126}} - \mathbf{45}_Z^* - \mathbf{1} - \mathbf{16}_1$

D13: $\mathbf{16}_1 - \mathbf{1} - \overline{\mathbf{126}} - \mathbf{45}_Z^* - \mathbf{1}'^* - \mathbf{1} - \mathbf{16}_3$

D31: $\mathbf{16}_3 - \mathbf{1} - \mathbf{1}'^* - \mathbf{45}_Z^* - \overline{\mathbf{126}} - \mathbf{1} - \mathbf{16}_1$

D13: $\mathbf{16}_1 - \mathbf{1} - \mathbf{45}_Z^* - \overline{\mathbf{126}} - \mathbf{1}'^* - \mathbf{1} - \mathbf{16}_3$

D31: $\mathbf{16}_3 - \mathbf{1} - \mathbf{1}'^* - \overline{\mathbf{126}} - \mathbf{45}_Z^* - \mathbf{1} - \mathbf{16}_1$

D13: $\mathbf{16}_1 - \mathbf{1} - \mathbf{45}_Z^* - \mathbf{1} - \mathbf{10}' - \mathbf{45}_X - \mathbf{16}_3$

D31: $\mathbf{16}_3 - \mathbf{45}_X - \mathbf{10}' - \mathbf{1} - \mathbf{45}_Z^* - \mathbf{1} - \mathbf{16}_1$

M11: $\mathbf{16}_1 - \mathbf{45}_X - \mathbf{1}' - \mathbf{45}_Z^* - \mathbf{45}_Z^* - \mathbf{45}_X - \mathbf{1}' - \overline{\mathbf{126}}' -$
$\mathbf{1}' - \mathbf{45}_X - \mathbf{45}_Z^* - \mathbf{45}_Z^* - \mathbf{1}' - \mathbf{45}_X - \mathbf{16}_1$

We thus have obtained the complete set of diagrams we shall consider for the evaluation of the mass matrices. Any additional diagrams for a given M_{Dirac} or M^R matrix element allowed by the $U(1)_F$ family symmetry are of higher-order and will lead to noticeably smaller contributions to that element than those arising from all the diagrams listed above.

IV. EVALUATION OF CONTRIBUTIONS to the MASS MATRICES

We now turn to the evaluation of the contributions to each matrix element at the SUSY GUT scale. The renormalizable 3-point couplings times VEVs for the $\mathbf{10}(5)$, $\mathbf{10}(\bar{5})$, $\mathbf{10}'(\bar{5}')$, $\overline{\mathbf{126}}(5)$, $\overline{\mathbf{126}}(\overline{45})$ and $\overline{\mathbf{126}}'(1)$ vertices contributing to M^U and M^N, M^D and M^E, M^D and M^E, M^U and M^N, M^D and M^E, and M^R, respectively, are labeled

$$g_{10}v_u, \quad g_{10}v_d, \quad g_{10'}v'_d, \quad g_{126}w_u, \quad g_{126}w_d, \quad g_{126'}w' \tag{4.1a}$$

We shall assume the superheavy fermions all get massive at the same mass scale, so each $\mathbf{1}$, $\mathbf{1}'$, $\mathbf{45}_X$ or $\mathbf{45}_Z$ vertex factor can be rescaled by the same propagator mass M according to

$$x \equiv g_{45_X}u_{45_X}/M, \; z \equiv g_{45_Z}u_{45_Z}/M, \; s \equiv g_1 u_1/M, \; s' \equiv g_{1'}u_{1'}/M \tag{4.1b}$$

where we have introduced a convenient short-hand notation. In order to accommodate CP violation, as noted earlier after (3.5b) we introduce the four phases

$$\phi_x, \quad \phi_z, \quad \phi_1, \quad \phi_{1'} \tag{4.1c}$$

As a result we are led to introduce 14 independent parameters in order to explain the 15 quark and lepton masses and 8 quark and lepton mixing parameters.

The contributions for each diagram then follow by moving along each fermion line and appending the above parameters together with the coupling coefficents spelled out in Table I. Alternatively, one can use the detailed computational procedure of Mohapatra and Sakita [23] which makes explicit use of the $SU(5)$ decompositions of the $SO(10)$ matrices and fields. We have used both procedures for a check in our calculations and both agree. In the expressions presented below, we have evaluated the Dirac $(\psi_L)^T C^{-1}(\psi^c)_L$ and Majorana ${(\psi^c)_L}^T C^{-1}(\psi^c)_L$ matrix elements .

Leading-Order Dirac Matrix Diagrams of (3.7a):

D33: $\mathbf{16}_3 - \mathbf{10} - \mathbf{16}_3$

$$M^U_{33} = M^N_{33} = g_{10} v_u, \qquad M^D_{33} = M^E_{33} = g_{10} v_d$$

D23: $\mathbf{16}_2 - \mathbf{10}' - \mathbf{16}_3$
D32: $\mathbf{16}_3 - \mathbf{10}' - \mathbf{16}_2$

$$M^D_{23} = M^D_{32} = M^E_{23} = M^E_{32} = g'_{10} v'_d$$

D22: $\mathbf{16}_2 - \overline{\mathbf{126}} - \mathbf{16}_2$ (4.2a)

$$(M^U_{22},\ M^N_{22}) = (1,\ -3) g_{126} w_u, \qquad (M^D_{22},\ M^E_{22}) = (1,\ -3) g_{126} w_d$$

D13: $\mathbf{16}_1 - \mathbf{45}_X - \mathbf{10}' - \mathbf{45}_X - \mathbf{16}_3$
D31: $\mathbf{16}_3 - \mathbf{45}_X - \mathbf{10}' - \mathbf{45}_X - \mathbf{16}_1$

$$M^D_{13} = M^D_{31} = M^E_{13} = M^E_{31} = -3 g_{10'} v'_d x^2 e^{2i\phi_x}$$

D12: $\mathbf{16}_1 - \mathbf{45}_Z - \mathbf{10}'^* - \mathbf{45}_Z - \mathbf{16}_2$
D21: $\mathbf{16}_2 - \mathbf{45}_Z - \mathbf{10}'^* - \mathbf{45}_Z - \mathbf{16}_1$

$$M^D_{12} = M^D_{21} = -4g_{10'}v'_d z^2 e^{2i\phi_z}, \qquad M^E_{12} = M^E_{21} = 0$$

Leading-Order Majorana Matrix Diagrams of (3.7b):

M33: $\mathbf{16}_3 - \mathbf{1} - \overline{\mathbf{126}}' - \mathbf{1} - \mathbf{16}_3$

$$M^R_{33} = g_{126'}w's^2e^{2i\phi_1}$$

M23: $\mathbf{16}_2 - \mathbf{1} - \mathbf{45}_X - \mathbf{1}' - \overline{\mathbf{126}}' - \mathbf{1} - \mathbf{16}_3$
M32: $\mathbf{16}_3 - \mathbf{1} - \overline{\mathbf{126}}' - \mathbf{1}' - \mathbf{45}_X - \mathbf{1} - \mathbf{16}_2$

$$M^R_{23} = M^R_{32} = 5g_{126'}w'xs^2s'e^{i(\phi_x+\phi_1+\phi_{1'})}$$

M22: $\mathbf{16}_2 - \mathbf{1} - \mathbf{45}_X - \mathbf{1}' - \overline{\mathbf{126}}' - \mathbf{1}' - \mathbf{45}_X - \mathbf{1} - \mathbf{16}_2$

$$M^R_{22} = 25g_{126'}w'(xss')^2e^{2i(\phi_x+\phi_1+\phi_{1'})}$$

M13: $\mathbf{16}_1 - \mathbf{45}_X - \mathbf{1}' - \mathbf{1} - \mathbf{45}_Z - \mathbf{1}' - \overline{\mathbf{126}}' - \mathbf{1} - \mathbf{16}_3$ (4.2b)
M31: $\mathbf{16}_3 - \mathbf{1} - \overline{\mathbf{126}}' - \mathbf{1}' - \mathbf{45}_Z - \mathbf{1} - \mathbf{1}' - \mathbf{45}_X - \mathbf{16}_1$

$$M^R_{13} = M^R_{31} = 30g_{126'}w'xzs^2s'^2e^{i(\phi_x+\phi_z+2\phi_1+2\phi_{1'})}$$

M12: $\mathbf{16}_1 - \mathbf{45}_X - \mathbf{1}' - \mathbf{1} - \mathbf{45}_Z - \mathbf{1}' - \overline{\mathbf{126}}' - \mathbf{1}' - \mathbf{45}_X - \mathbf{1} - \mathbf{16}_2$
M21: $\mathbf{16}_2 - \mathbf{1} - \mathbf{45}_X - \mathbf{1}' - \overline{\mathbf{126}}' - \mathbf{1}' - \mathbf{45}_Z - \mathbf{1} - \mathbf{1}' - \mathbf{45}_X - \mathbf{16}_1$

$$M^R_{12} = M^R_{21} = 150g_{126'}w'x^2zs^2s'^3e^{i(2\phi_x+\phi_z+2\phi_1+3\phi_{1'})}$$

M11: $\mathbf{16}_1 - \mathbf{45}_X - \mathbf{1}' - \mathbf{1} - \mathbf{45}_Z - \mathbf{1}' - \overline{\mathbf{126}}' - \mathbf{1}' - \mathbf{45}_Z - \mathbf{1} - \mathbf{1}' - \mathbf{45}_X - \mathbf{16}_1$

$$M^R_{11} = 900g_{126'}w'(xzss'^2)^2e^{2i(\phi_x+\phi_z+\phi_1+2\phi_{1'})}$$

Higher-Order Diagrams listed in (3.7c) from Minimal Set:

D11: $\mathbf{16}_1 - \mathbf{45}_X - \mathbf{1}' - \mathbf{1} - \mathbf{10}' - \mathbf{1} - \mathbf{1}' - \mathbf{45}_X - \mathbf{16}_1$

$$M^D_{11} = M^E_{11} = -3g_{10'}v'_d(xss')^2e^{2i(\phi_x+\phi_1+\phi_{1'})}$$

D22: $\mathbf{16}_2\ -\mathbf{45}_Z\ -\mathbf{10}'^{*} - \mathbf{1}'^{*} - \mathbf{16}_2, \qquad \mathbf{16}_2\ -\mathbf{1}'^{*} - \mathbf{10}'^{*} - \mathbf{45}_Z\ -\mathbf{16}_2$

$$M^D_{22} = M^E_{22} = -3g_{10'}v'_d zs'e^{i(\phi_z-\phi_{1'})} \tag{4.2c}$$

M23: $\mathbf{16}_2\ -\mathbf{45}^*_X\ -\mathbf{1}' - \mathbf{1} - \mathbf{45}_Z\ -\mathbf{1}' - \overline{\mathbf{126}}' - \mathbf{1} - \mathbf{16}_3$

M32: $\mathbf{16}_3\ -\mathbf{1} - \overline{\mathbf{126}}' - \mathbf{1}' - \mathbf{45}_Z\ -\mathbf{1} - \mathbf{1}' - \mathbf{45}^*_X\ -\mathbf{16}_2$

$$M^R_{23} = M^R_{32} = 30g_{126'}w'xzs^2s'^2e^{i(-\phi_x+\phi_z+2\phi_1+2\phi_{1'})}$$

Higher-Order Diagrams of (3.7d) from the Expanded Set:

D11: $\mathbf{16}_1\ -\mathbf{1} - \overline{\mathbf{126}} - \mathbf{1} - \mathbf{1}' - \mathbf{45}_X\ -\mathbf{16}_1, \qquad \mathbf{16}_1\ -\mathbf{45}_X\ -\mathbf{1}' - \mathbf{1} - \overline{\mathbf{126}} - \mathbf{1} - \mathbf{16}_1$

$$(M^U_{11},\ M^N_{11}) = (2,\ -6)g_{126}w_u xs^2s'e^{i(\phi_x+2\phi_1+\phi_{1'})}$$

$$(M^D_{11},\ M^E_{11}) = (-2,\ 6)g_{126}w_d xs^2s'e^{i(\phi_x+2\phi_1+\phi_{1'})}$$

D11: $\mathbf{16}_1\ -\mathbf{1} - \mathbf{45}^*_Z\ -\mathbf{1} - \mathbf{1}' - \mathbf{1} - \overline{\mathbf{126}} - \mathbf{1} - \mathbf{16}_1,$
$\mathbf{16}_1\ -\mathbf{1} - \overline{\mathbf{126}} - \mathbf{1} - \mathbf{1}' - \mathbf{1} - \mathbf{45}^*_Z\ -\mathbf{1} - \mathbf{16}_1$

$$(M^U_{11},\ M^N_{11}) = (3,\ -9)g_{126}w_u zs^4s'e^{i(-\phi_z+4\phi_1+\phi_{1'})}$$

$$(M^D_{11},\ M^E_{11}) = (-3,\ 9)g_{126}w_d zs^4s'e^{i(-\phi_z+4\phi_1+\phi_{1'})}$$

D11: $\mathbf{16}_1\ -\mathbf{1} - \mathbf{45}^*_Z\ -\mathbf{1} - \mathbf{1}' - \mathbf{1} - \mathbf{10}' - \mathbf{1} - \mathbf{1}' - \mathbf{45}_X\ -\mathbf{16}_1,$
$\mathbf{16}_1\ -\mathbf{45}_X\ -\mathbf{1}' - \mathbf{1} - \mathbf{10}' - \mathbf{1} - \mathbf{1}' - \mathbf{1} - \mathbf{45}^*_Z\ -\mathbf{1} - \mathbf{16}_1$

$$(M^D_{11},\ M^E_{11}) = (-7,\ -3)g_{10'}v'_d xzs^4s'^2e^{i(\phi_x-\phi_z+4\phi_1+2\phi_{1'})}$$

D12: $\mathbf{16}_1\ -\mathbf{1} - \overline{\mathbf{126}} - \mathbf{1} - \mathbf{1}' - \mathbf{45}^*_X\ -\mathbf{16}_2$

$$(M^U_{12},\ M^N_{12}) = (1,\ -15)g_{126}w_u xs^2s'e^{i(-\phi_x+2\phi_1+\phi_{1'})}$$

$$M^D_{12} = M^E_{12} = -3g_{126}w_d xs^2s'e^{i(-\phi_x+2\phi_1+\phi_{1'})}$$

D21: $\mathbf{16}_2\ -\mathbf{45}^*_X\ -\mathbf{1}' - \mathbf{1} - \overline{\mathbf{126}} - \mathbf{1} - \mathbf{16}_1$

$$(M^U_{21},\ M^N_{21}) = (1,\ 9)g_{126}w_u xs^2s'e^{i(-\phi_x+2\phi_1+\phi_{1'})}$$

$$(M^D_{21},\ M^E_{21}) = (1,\ 9)g_{126}w_d xs^2s'e^{i(-\phi_x+2\phi_1+\phi_{1'})}$$

D12: $\mathbf{16}_1\ -\mathbf{1} - \overline{\mathbf{126}} - \mathbf{45}^*_Z\ -\mathbf{45}_X\ -\mathbf{1} - \mathbf{16}_2$

$$(M^U_{12},\ M^N_{12}) = (2,\ -90)g_{126}w_u xzs^2e^{i(\phi_x-\phi_z+2\phi_1)}$$

$$M^D_{12} = 12g_{126}w_d xzs^2e^{i(\phi_x-\phi_z+2\phi_1)}, \qquad M^E_{12} = 0$$

D21: $\mathbf{16}_2 - \mathbf{1} - \mathbf{45}_X - \mathbf{45}_Z^* - \overline{\mathbf{126}} - \mathbf{1} - \mathbf{16}_1$

$$(M_{21}^U,\ M_{21}^N) = (1,\ -27) g_{126} w_u xzs^2 e^{i(\phi_x - \phi_z + 2\phi_1)}$$
$$(M_{21}^D,\ M_{21}^E) = (1,\ -27) g_{126} w_d xzs^2 e^{i(\phi_x - \phi_z + 2\phi_1)}$$

D12: $\mathbf{16}_1 - \mathbf{1} - \mathbf{45}_Z^* - \overline{\mathbf{126}} - \mathbf{45}_X - \mathbf{1} - \mathbf{16}_2$ (4.2d)

$$(M_{12}^U,\ M_{12}^N) = (1,\ 45) g_{126} w_u xzs^2 e^{i(\phi_x - \phi_z + 2\phi_1)}$$
$$(M_{12}^D,\ M_{12}^E) = (-3,\ 9) g_{126} w_d xzs^2 e^{i(\phi_x - \phi_z + 2\phi_1)}$$

D21: $\mathbf{16}_2 - \mathbf{1} - \mathbf{45}_X - \overline{\mathbf{126}} - \mathbf{45}_Z^* - \mathbf{1} - \mathbf{16}_1$

$$(M_{21}^U,\ M_{21}^N) = (2,\ 54) g_{126} w_u xzs^2 e^{i(\phi_x - \phi_z + 2\phi_1)}$$
$$M_{21}^D = -4 g_{126} w_d xzs^2 e^{i(\phi_x - \phi_z + 2\phi_1)}, \qquad M_{21}^E = 0$$

D13: $\mathbf{16}_1 - \mathbf{1} - \overline{\mathbf{126}} - \mathbf{45}_Z^* - \mathbf{1}'^* - \mathbf{1} - \mathbf{16}_3$

$$(M_{13}^U,\ M_{13}^N) = (2,\ -18) g_{126} w_u zs^2 s' e^{i(-\phi_z + 2\phi_1 - \phi_{1'})}$$
$$M_{13}^D = -4 g_{126} w_d zs^2 s' e^{i(-\phi_z + 2\phi_1 - \phi_{1'})}, \qquad M_{13}^E = 0$$

D31: $\mathbf{16}_3 - \mathbf{1} - \mathbf{1}'^* - \mathbf{45}_Z^* - \overline{\mathbf{126}} - \mathbf{1} - \mathbf{16}_1$

$$(M_{31}^U,\ M_{31}^N) = (1,\ 9) g_{126} w_u zs^2 s' e^{i(-\phi_z + 2\phi_1 - \phi_{1'})}$$
$$(M_{31}^D,\ M_{31}^E) = (1,\ 9) g_{126} w_d zs^2 s' e^{i(-\phi_z + 2\phi_1 - \phi_{1'})}$$

D13: $\mathbf{16}_1 - \mathbf{1} - \mathbf{45}_Z^* - \overline{\mathbf{126}} - \mathbf{1}'^* - \mathbf{1} - \mathbf{16}_3$

$$(M_{13}^U,\ M_{13}^N) = (1,\ 9) g_{126} w_u zs^2 s' e^{i(-\phi_z + 2\phi_1 - \phi_{1'})}$$
$$(M_{13}^D,\ M_{13}^E) = (1,\ 9) g_{126} w_d zs^2 s' e^{i(-\phi_z + 2\phi_1 - \phi_{1'})}$$

D31: $\mathbf{16}_3 - \mathbf{1} - \mathbf{1}'^* - \overline{\mathbf{126}} - \mathbf{45}_Z^* - \mathbf{1} - \mathbf{16}_1$

$$(M_{31}^U,\ M_{31}^N) = (2,\ -18) g_{126} w_u zs^2 s' e^{i(-\phi_z + 2\phi_1 - \phi_{1'})}$$
$$M_{31}^D = -4 g_{126} w_d zs^2 s' e^{i(-\phi_z + 2\phi_1 - \phi_{1'})}, \qquad M_{31}^E = 0$$

D13: $\mathbf{16}_1 - \mathbf{1} - \mathbf{45}_Z^* - \mathbf{1} - \mathbf{10}' - \mathbf{45}_X - \mathbf{16}_3$

$$M_{13}^D = M_{13}^E = -3 g_{10'} v_d' xzs^2 e^{i(\phi_x - \phi_z + 2\phi_1)}$$

D31: $\mathbf{16}_3\ -\mathbf{45}_X\ -\mathbf{10}' - \mathbf{1} - \mathbf{45}^*_Z\ -\mathbf{1} - \mathbf{16}_1$

$$M^D_{31} = -4g_{10'}v'_d xzs^2 e^{i(\phi_x - \phi_z + 2\phi_1)}, \qquad M^E_{31} = 0$$

M11: $\mathbf{16}_1\ -\mathbf{45}_X\ -\mathbf{1}' - \mathbf{45}^*_Z\ -\mathbf{45}^*_Z\ -\mathbf{45}_X\ -\mathbf{1}' - \overline{\mathbf{126}}'-$
$\mathbf{1}' - \mathbf{45}_X\ -\mathbf{45}^*_Z\ -\mathbf{45}^*_Z\ -\mathbf{1}' - \mathbf{45}_X\ -\mathbf{16}_1$

$$M^R_{11} = (900)^2 g_{126'} w'(xzs')^4 e^{4i(\phi_x - \phi_z + \phi_{1'})}$$

An interesting observation which can be drawn from the Majorana contributions in (4.2b) is that the matrix in leading order has a geometrical texture as given in (2.4c) with

$$M^R_{22} \simeq \frac{5}{6}\sqrt{M^R_{11}M^R_{33}} \tag{4.3}$$

provided $x \simeq z$. In fact, this observation served as an important guide in our construction of the Majorana neutrino matrix and suggested the relative roles played by the $\mathbf{45}_X$ and $\mathbf{45}_Z$ Higgs fields.

V. QUANTITATIVE RESULTS for the $SO(10) \times U(1)_F$ MODEL

Finally we attempt to select a set of values for the 14 input parameters of (4.1a,b,c) which will accurately reproduce the input data in (2.1a,b) and (2.3a,b) used for our bottom-up approach. As noted earlier, the minimal set of superheavy fermions and their mirror partners found in (3.8b) yield unsatisfactory results: $m_u = m_{\nu_e} = 0$, $m_e = 0.006$ MeV and $m_{\nu_\mu} = m_{\nu_\tau} = 0.089$ eV. The problem can be traced to the zero or tiny values of D11. By expanding the set of superheavy fermions to include those in (3.8c), on the other hand, excellent results can be found as shown below.

One particularly good numerical choice for the parameters at the SUSY GUT scale is given by

$$
\begin{aligned}
g_{10}v_u &= 120.3, & g_{10}v_d &= 2.46, & g_{10'}v'_d &= 0.078\ \text{GeV} \\
g_{126}w_u &= 0.314, & g_{126}w_d &= -0.037, & g_{126'}w' &= 0.8\times 10^{16}\ \text{GeV} \\
g_{45_X}u_{45_X}/M &= 0.130, & g_{45_Z}u_{45_Z}/M &= 0.165, & g_1u_1/M &= 0.56, \quad g_{1'}u'_1/M = -0.026 \\
\phi_x &= 35^o, & \phi_z &= \phi_1 = \phi_{1'} = -5^o
\end{aligned}
\tag{5.1}
$$

which reduces the number of independent parameters from 14 to 12. In fact, the only large phase angle is that for ϕ_x. As seen from (4.2a), this is in agreement with our earlier conclusion from the bottom-up phenomenological results [8] that essentially only the Dirac D13 and D31 matrix elements are complex. The mass matrices at the SUSY GUT scale are then numerically equal to

$$
M^U = \begin{pmatrix} -0.0010-0.0001i & 0.0053+0.0034i & -0.0013 \\ 0.0053+0.0034i & 0.314 & 0 \\ -0.0013 & 0 & 120.3 \end{pmatrix} \tag{5.2a}
$$

$$
M^D = \begin{pmatrix} -0.0001 & -0.0104+0.0004i & -0.0029-0.0045i \\ -0.0077+0.0018i & -0.036 & 0.078 \\ -0.0033-0.0048i & 0.078 & 2.460 \end{pmatrix} \tag{5.2b}
$$

$$
M^N = \begin{pmatrix} 0.0030+0.0003i & -0.079-0.051i & 0.0038 \\ 0.048+0.031i & -0.942 & 0 \\ 0.0038 & 0 & 120.3 \end{pmatrix} \tag{5.2c}
$$

$$
M^E = \begin{pmatrix} 0.0004 & -0.0020-0.0010i & -0.0023-0.0045i \\ 0.0060+0.0031i & 0.112 & 0.078 \\ -0.0009-0.0037i & 0.078 & 2.460 \end{pmatrix} \tag{5.2d}
$$

$$
M^R = \begin{pmatrix} (-.069+.640i)\times 10^9 & (-.141-.119i)\times 10^{11} & (.108+.019i)\times 10^{13} \\ (-.141-.119i)\times 10^{11} & (.461+.549i)\times 10^{12} & (-.393-.155i)\times 10^{14} \\ (.108+.019i)\times 10^{13} & (-.393-.155i)\times 10^{14} & (.247-.044i)\times 10^{16} \end{pmatrix} \tag{5.2e}
$$

in units of GeV. By using the seesaw formula [16], we find for the light neutrino matrix at

the SUSY GUT scale

$$
\begin{aligned}
M^{N_{eff}} &\simeq -M^N (M^R)^{-1} M^{N^T} \\
&= \begin{pmatrix} (.027 - .238i)\times 10^{-3} & (-.109 - .199i)\times 10^{-2} & (-.037 + .512i)\times 10^{-2} \\ (-.109 - .199i)\times 10^{-2} & (-.232 - .088i)\times 10^{-1} & (.258 + .435i)\times 10^{-1} \\ (-.037 + .512i)\times 10^{-2} & (.258 + .435i)\times 10^{-1} & -.001 - .112i \end{pmatrix}
\end{aligned} \tag{5.2f}
$$

in electron-Volts. Again we emphasize the Dirac mass matrix elements appear in the form $\psi^T_{iL} C^{-1} M (\psi^c)_{jL}$, while the Majorana matrix elements refer to $(\psi^c)^T_{iL} C^{-1} M (\psi^c)_{jL}$ with ψ_{iL} and $(\psi^c)_{jL}$ each a member of one of the three families of **16**'s. Identical contributions also arise from the transposed Dirac matrices and the right-handed Majorana matrix. As such, the true Yukawa couplings G_Y are just half the values of the g_Y's appearing in (4.1a,b).

The masses at the GUT scale can then be found by calculating the eigenvalues of the Hermitian product $MM^\dagger$ in each case, while the mixing matrices V_{CKM} and V_{lept} can be calculated with the projection operator technique of Jarlskog [24]. After evolving these quantities to the low scale, we find in the quark sector

$$
\begin{aligned}
m_u(1\text{GeV}) &= 5.0\ (5.1)\ \text{MeV}, & m_d(1\text{GeV}) &= 7.9\ (8.9)\ \text{MeV} \\
m_c(m_c) &= 1.27\ (1.27)\ \text{GeV}, & m_s(1\text{GeV}) &= 169\ (175)\ \text{MeV} \\
m_t(m_t) &= 150\ (165)\ \text{GeV}, & m_b(m_b) &= 4.09\ (4.25)\ \text{GeV}
\end{aligned} \tag{5.3a}
$$

where we have indicated the preferred values in parentheses. The mixing matrix is given by

$$
V_{CKM} = \begin{pmatrix} 0.972 & 0.235 & 0.0037 e^{-i124^o} \\ -0.235 & 0.971 & 0.041 \\ \begin{matrix} 0.012 \\ -0.003i \end{matrix} & \begin{matrix} -0.039 \\ -0.001i \end{matrix} & 0.999 \end{pmatrix} \tag{5.3b}
$$

Note that $V_{cb} = 0.041$ and $|V_{ub}/V_{cb}| = 0.090$ with the CP-violating phase $\delta = 124^o$, while $m_d/m_u = 1.59$ and $m_s/m_d = 21.3$, cf. [12, 13]. These results should be compared with our central starting input values given in (2.1a,b).

In the lepton sector we obtain

$$\begin{aligned} m_{\nu_e} &= 0.10\ (?) \times 10^{-4}\ \mathrm{eV}, & m_e &= 0.43\ (0.511)\ \mathrm{MeV} \\ m_{\nu_\mu} &= 0.29\ (0.25) \times 10^{-2}\ \mathrm{eV}, & m_\mu &= 103\ (105.5)\ \mathrm{MeV} \\ m_{\nu_\tau} &= 0.12\ (0.10)\ \mathrm{eV}, & m_\tau &= 1.777\ (1.777)\ \mathrm{GeV} \end{aligned} \tag{5.4a}$$

and

$$V_{lept} = \begin{pmatrix} 0.998 & 0.049 & 0.039 e^{-i121^o} \\ -0.036 & 0.875 & 0.483 \\ \begin{matrix} 0.042 \\ -0.037i \end{matrix} & \begin{matrix} -0.482 \\ -0.002i \end{matrix} & 0.875 \end{pmatrix} \tag{5.4b}$$

which should be compared with the input values in (2.3a,b). The heavy Majorana neutrino masses are

$$M_1^R = 0.63 \times 10^9\ \mathrm{GeV}, \qquad M_2^R = 0.37 \times 10^{11}\ \mathrm{GeV}, \qquad M_3^R = 0.25 \times 10^{16}\ \mathrm{GeV} \tag{5.4c}$$

The neutrino masses and mixings are in the correct ranges to explain the nonadiabatic solar neutrino depletion with small mixing [5] and the atmospheric neutrino depletion with large mixing [6]:

$$\begin{aligned} \delta m_{12}^2 &= 8.5 \times 10^{-6}\ \mathrm{eV}^2, & \sin^2 2\theta_{12} &= 0.0062 \\ \delta m_{23}^2 &= 1.4 \times 10^{-2}\ \mathrm{eV}^2, & \sin^2 2\theta_{23} &= 0.71 \end{aligned} \tag{5.5}$$

For our analysis, the SUSY GUT scale at which the gauge and Yukawa couplings unify was chosen to be $\Lambda = 1.2 \times 10^{16}$ GeV. From (3.4a) and (5.2a,b,c,d) we find that $g_{10} = 0.69$. It is interesting to note that if we equate the $SO(10)$-breaking and lepton number-breaking VEV, w', with Λ, we find $g_{126'} = 0.67 \simeq g_{10}$. Taking into account the remark following (5.2e), we note the true Yukawa couplings are $G_{10} \simeq G_{126'} \simeq 0.33$. If we further equate $g_1 = g_{10} \simeq g_{126'}$, and $u_1 = \Lambda$ for the $U(1)_F$-breaking VEV, we find $M = 1.5 \times 10^{16}$ GeV for the masses of the superheavy fermions which condense with their mirrors. These values are all very reasonable.

The $\mathbf{45}_X$ and $\mathbf{45}_Z$ VEVs appear at nearly the same scale, 2.8×10^{15} and 3.5×10^{15} GeV respectively, if one assumes the same Yukawa coupling as above. On the other hand, if these VEVs appear at the unification scale Λ the corresponding Yukawa couplings are smaller than those found above. In either case, a consequence of their non-orthogonal breakings is that $SU(5)$ is broken down to $SU(3)_c \times SU(2)_L \times U(1)_Y$ at the scale in question. No further breaking is required until the electroweak scale and the SUSY-breaking scale are reached.

VI. SUMMARY

Our starting point for this research has been based on the results obtained from a bottom-up approach proposed previously by us to obtain mass matrices at the SUSY GUT scale based on a complete set of data inputted at the low scales. In particular we have used the known quark and charged lepton masses and CKM mixing matrix together with the neutrino masses and mixings based on particular neutrino scenarios. The masses and mixing matrices were evolved to the SUSY GUT scale where the mass matrices can be constructed by use of Sylvester's theorem. By varying the bases and the signs of the mass eigenvalues, we looked for simple textures for the mass matrices such that each matrix element involved as few $SO(10)$ Higgs representations as possible. The neutrino scenario examined which appeared to yield the simplest structure involved the MSW nonadiabatic depletion of the solar electron-neutrinos together with the observed depletion of atmospheric muon-neutrinos by oscillations into tau-neutrinos.

In this paper we have constructed an $SO(10) \times U(1)_F$ model of the Yukawa interactions which neatly reproduces the desired $SO(10)$ textures for the quark and lepton mass matrices for this preferred neutrino scenario. The observed features include the following:

(i) The Abelian $U(1)_F$ family symmetry group singles out a rather simple set of tree diagrams which determines the texture of the generic Dirac and Majorana mass matrices,

while the $SO(10)$ group relates corresponding matrix elements of the up, down, neutrino and charged lepton Dirac matrices to each other.

(ii) The dominant second and third family Yukawa interactions are renormalizable and arise through couplings with Higgs in the $\mathbf{10}$, $\mathbf{10}'$ and $\overline{\mathbf{126}}$ representations of $SO(10)$. The remaining Yukawa interactions are of higher order and require couplings of Higgs in the $\overline{\mathbf{126}}'$, $\mathbf{1}$, $\mathbf{1}'$, $\mathbf{45}_X$ and $\mathbf{45}_Z$ representations which acquire VEVs near the SUSY GUT scale.

(iii) The Higgs which acquire high scale VEVs break the $SO(10)\times U(1)_F$ symmetry down to the $SU(3)_c\times SU(2)_L\times U(1)_Y$ standard model symmetry in two stages through the $SU(5)$ subgroup.

(iv) Although this non-minimal supersymmetric model involves several Higgs representations, the runnings of the Yukawa couplings from the GUT scale to the low-energy SUSY-breaking scale are controlled mainly by the contributions from the $\mathbf{10}$, as in the minimal supersymmetric standard model.

(v) The complete set of low scale VEVs which contribute to the fermion masses are $\mathbf{10}(5)$, $\mathbf{10}(\bar{5})$, $\mathbf{10}'(\bar{5}')$, $\overline{\mathbf{126}}(5)$ and $\overline{\mathbf{126}}(\overline{45})$ in the $SO(10)(SU(5))$ notation. These Higgs correspond to the minimum number required in $SO(10)$ models which lead to the successful Georgi - Jarlskog relations [3]. Most of these models, however, do not include neutrino mass matrices.

(vi) In terms of 12 input parameters, 15 masses (including the heavy Majorana masses) and 8 mixing parameters emerge. The Yukawa couplings and the Higgs VEVs are numerically feasible and successfully correlate all the quark and lepton masses and mixings in the scenario which incorporates the nonadiabatic solar neutrino and atmospheric neutrino depletion effects.

(vii) The right-handed Majorana neutrino matrix has a nearly geometrical texture leading to heavy Majorana neutrino masses spread over seven orders of magnitude as given in (5.4c). In fact, it is the highly geometrical structure of the Majorana matrix which accounts

for the nearly maximal mixing of the ν_μ and ν_τ, rather than sizable mixing in the Dirac sector [25].

With the model as presented, the $U(1)_F$ current is anomalous, since the $[U(1)_F]^3$ triangle anomaly does not vanish. It is possible to cancel this anomaly, however, by the addition of two $SO(10)$ singlet neutral fermions, n_L and $(n^c)_L$, both with $U(1)_F$ charges of -12. By introducing another Higgs singlet representation which develops a GUT scale VEV, one can arrange that one of the new neutrinos remains massless while the other becomes superheavy. Alternatively, it is possible to cancel such an anomaly through the Green-Schwarz mechanism [22] provided the model can be derived from string theory.

Studies are underway to examine what effects small mixings of such a light sterile neutrino with the three families of light neutrinos will have on the neutrino spectrum and will be reported elsewhere. Work is also underway to construct a superpotential for the model presented here.

The authors thank the Fermilab Theoretical Physics Department for the opportunity to participate in the 1994 and 1995 Summer Visitor Programs during which time the research reported here was initiated and completed. Useful discussions with Joseph Lykken and Rabindra Mohapatra about various aspects of this work are gratefully acknowledged. This research was supported in part by the U. S. Department of Energy.

References

[1] U. Amaldi, W. de Boer, and H. Furstenau, Phys. Lett. B **260**, 447 (1991); J. Ellis, S. Kelley, and D. V. Nanopoulos, ibid. **260**, 131 (1991); P. Langacker and M. Luo, Phys. Rev. D **44**, 817 (1991).

[2] S. Weinberg, Proceedings of Festschrift in honor of I. I. Rabi, New York Academy of Science; F. Wilczek and A. Zee, Phys. Lett. B **70**, 418 (1977), erratum, ibid. B **72**, 504 (1978); H. Fritzsch, Phys. Lett. B **70**, 436 (1977).

[3] H. Georgi and C. Jarlskog, Phys. Lett. B **86**, 297 (1979); J. A. Harvey, P. Ramond, and D. B. Reiss, Phys. Lett. B **92**, 309 (1980); H. Arason, D. Castanño, B. Keszthelyi, S. Mikaelian, E. Piard, P. Ramond, and B. Wright, Phys. Rev. Lett. **67**, 2933 (1991); Phys. Rev. D **46**, 3945 (1992); S. Dimopoulos, L. J. Hall, and S. Raby, Phys. Rev. Lett. **68**, 1984 (1992); Phys. Rev. D **45**, 4192 (1992); **46**, R4793 (1992); **47**, R3702 (1993); G. F. Giudice, Mod. Phys. Lett. A **7**, 2429 (1992); H. Arason, D. Castanño, P. Ramond and E. Piard, Phys. Rev. D **47**, 232 (1993); P. Ramond, R. G. Roberts, and G. G. Ross, Nucl. Phys. **B406**, 19 (1993); A. Kusenko and R. Shrock, Phys. Rev. D **49**, 4962 (1994); K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. **74**, 2418 (1995).

[4] S. P. Mikheyev and A. Yu Smirnov, Yad Fiz. **42**, 1441 (1985) [Sov. J. Nucl. Phys. **42**, 913 (1986)]; Zh. Eksp. Teor. Fiz. **91**, 7 (1986) [Sov. Phys. JETP **64**, 4 (1986)]; Nuovo Cimento **9C**, 17 (1986); L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978); **20**, 2634 (1979).

[5] R. Davis et al., Phys. Rev. Lett. **20**, 1205 (1968); in *Neutrino '88*, ed. J. Schnepp et al. (World Scientific, 1988); K. Hirata et al., Phys. Rev. Lett. **65**, 1297, 1301 (1990);

P. Anselmann et al., Phys. Lett. B **327**, 377, 390 (1994); Dzh. N. Abdurashitov et al., Phys. Lett. B **328**, 234 (1994).

[6] K. S. Hirata et al., Phys. Lett. B **280**, 146 (1992); and **283**, 446 (1992); R. Becker-Szendy et al., Phys. Rev. Lett. **69**, (1992) and Phys. Rev. D **46**, 3720 (1992); W. W. M. Allison et al., Report No. ANL-HEP-CP-93-32; Y. Fukuda et al., Phys. Lett. B **335**, 237 (1994).

[7] Q. Shafi and F. Stecker, Phys. Rev. Lett. **53**, 1292 (1984); Ap. J. **347**, 575 (1989).

[8] C. H. Albright and S. Nandi, Phys. Rev. Lett. **73**, 930 (1994); Phys. Rev. D **52**, 410 (1995); Report No. Fermilab-PUB-94/119-T and OSU Report No. 289 (to be published); C. H. Albright, in Proceedings of the Yukawa Conference held at the University of Florida, February 1994 (International Press, Cambridge, Massachusetts, 1995).

[9] F. R. Grantmakher, *Theory of Matrices*, (Chelsa Publishing Company, New York, 1959).

[10] C. H. Albright and S. Nandi, Report no. FERMILAB-PUB-95/107-T and OSU Report No. 303, to be published.

[11] S. Dimopoulos, Phys. Lett. B **129**, 417 (1983).

[12] S. Naculich, Phys. Rev. D **48**, 5293 (1993).

[13] Particle Data Group, M. Aguilar-Benitez et al., Phys. Rev. D **50**, 1173 (1994).

[14] J. Gasser and H. Leutwyler, Phys. Rep. C **87**, 77 (1982).

[15] A. Kusenko, Phys. Lett. B **284**, 390 (1992).

[16] M. Gell-Mann, P. Ramond, and R. Slansky, in *Supersymmetry*, edited by P. Van Nieuwenhuizen and D. Z. Freedman (North-Holland, Amsterdam, 1979); T. Yanagida, Prog. Theor. Phys. **B 315**, 66 (1978).

[17] E. H. Lemke, Mod. Phys. Lett. A **7**, 1175 (1992).

[18] The form of M^R presented here differs somewhat from that in Ref. [1], for more recent data on the atmospheric depletion effect was taken into account.

[19] For recent use of $U(1)_F$ symmetry to generate patterns of fermion mass matrices, see L. Ibanez and G. G. Ross, Phys. Lett. B **332**, 100 (1994); P. Binetruy and P. Ramond, Phys. Lett. B **350**, 49 (1995); H. Dreiner, G. K. Leontaris , S. Lola, and G. G. Ross, Nucl. Phys. B **436**, 461 (1995); V. Jain and R. Shrock, Report No. ITP-SB-94-55.

[20] For reviews, cf. P. Fayet and S. Ferrara, Phys. Rep. C **32**, 249 (1977); P. Van Niewenhuizen, ibid. **68**, 189 (1981); J. Wess and J. Bagger, *Supersymmetry and Supergravity*, Princeton University Press, Princeton, New Jersey (1983); P. Nath, R. Arnowitt and A. H. Chamseddine, Applied N = 1 Supergravity, World Scientific, Singapore (1983); H. P. Nilles, Phys. Rep. C **110**, 1 (1984).

[21] Jorge L. Lopez and D. V. Nanopoulos, Proceedings of 15th Johns Hopkins Workshop, (1991), p. 277.

[22] M. B. Green and J. H. Schwarz, Phys. Lett. B **149**, 117 (1984); M. Dine, N. Seiberg and E. Witten, Nucl. Phys. B **289**, 589 (1987).

[23] R. N. Mohapatra and B. Sakita, Phys. Rev. D **21**, 1062 (1980).

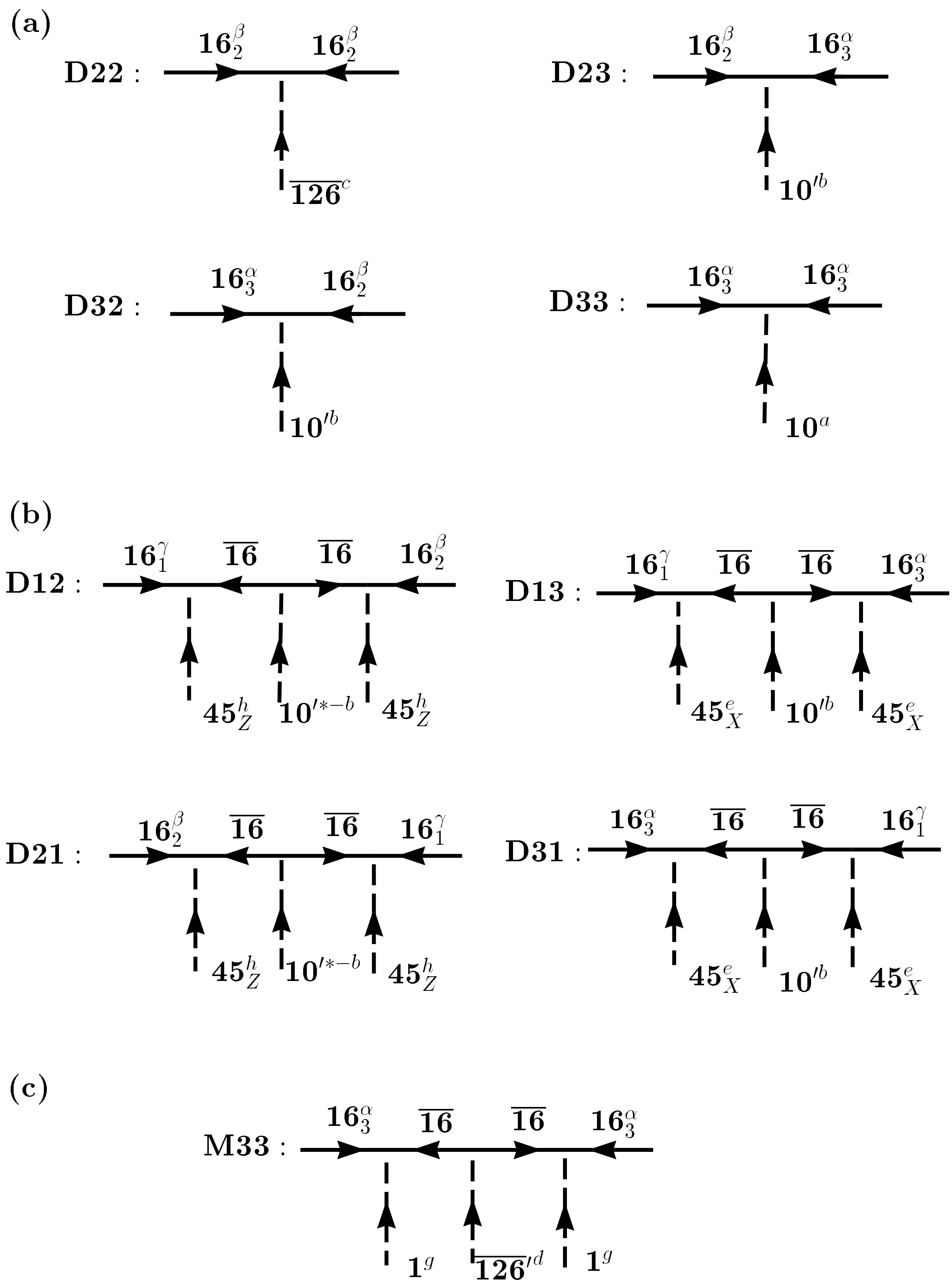
[24] C. Jarlskog, Phys. Rev. D **35**, 1685 (1987); **36** 2138 (1987); C. Jarlskog and A. Kleppe, Nucl. Phys. **B286**, 245 (1987).

[25] An early observation of this effect in a Monte Carlo analysis was reported in C. H. Albright, Phys. Rev. D **R725**, (1992). An analytical analysis was subsequently reported by A. Yu. Smirnov, Phys. Rev. D **48**, 3264 (1993).

SU(5) Assignments	VEV Directions $\mathbf{45}_X$	$\mathbf{45}_Y$	$\mathbf{45}_Z$	Flipped SU(5) Assignments
$u,\ d$	1	1	1	$d,\ u$
u^c	1	- 4	2	d^c
d^c	- 3	2	- 4	u^c
$\nu,\ \ell$	- 3	- 3	- 3	$\ell,\ \nu$
ν^c	5	0	6	e^c
e^c	1	6	0	ν^c

Table I. Couplings of the **45** VEVs to states in the **16**.

Fig. 1. Tree-level diagrams for the (a) renormalizable and (b) leading-order nonrenormalizable contributions to the generic Dirac mass matrix and for the (c) 33 element of the Majorana mass matrix.


(a)
D22 :
$\mathbf{16}_2^\beta$
$\mathbf{16}_2^\beta$
$\overline{\mathbf{126}}^c$
D23 :
$\mathbf{16}_2^\beta$
$\mathbf{16}_3^\alpha$
$\mathbf{10}'^b$
D32 :
$\mathbf{16}_3^\alpha$
$\mathbf{16}_2^\beta$
$\mathbf{10}'^b$
D33 :
$\mathbf{16}_3^\alpha$
$\mathbf{16}_3^\alpha$
$\mathbf{10}^a$
(b)
D12 :
$\mathbf{16}_1^\gamma$
$\overline{\mathbf{16}}$
$\overline{\mathbf{16}}$
$\mathbf{16}_2^\beta$
$\mathbf{45}_Z^h$
$\mathbf{10}'^{*-b}$
$\mathbf{45}_Z^h$
D13 :
$\mathbf{16}_1^\gamma$
$\overline{\mathbf{16}}$
$\overline{\mathbf{16}}$
$\mathbf{16}_3^\alpha$
$\mathbf{45}_X^e$
$\mathbf{10}'^b$
$\mathbf{45}_X^e$
D21 :
$\mathbf{16}_2^\beta$
$\overline{\mathbf{16}}$
$\overline{\mathbf{16}}$
$\mathbf{16}_1^\gamma$
$\mathbf{45}_Z^h$
$\mathbf{10}'^{*-b}$
$\mathbf{45}_Z^h$
D31 :
$\mathbf{16}_3^\alpha$
$\overline{\mathbf{16}}$
$\overline{\mathbf{16}}$
$\mathbf{16}_1^\gamma$
$\mathbf{45}_X^e$
$\mathbf{10}'^b$
$\mathbf{45}_X^e$
(c)
M33 :
$\mathbf{16}_3^\alpha$
$\overline{\mathbf{16}}$
$\overline{\mathbf{16}}$
$\mathbf{16}_3^\alpha$
$\mathbf{1}^g$
$\overline{\mathbf{126}}'^d$
$\mathbf{1}^g$